Matrix notation

A matrix with m rows and n columns is called an <u>m x n matrix</u>.

An m x 1 matrix is a <u>column matrix</u>. $\left.\overbrace{\begin{bmatrix} * \\ * \\ \vdots \\ * \end{bmatrix}}^{1}\right\} m$

A 1 x n matrix is a <u>row matrix</u>. $\underbrace{\begin{bmatrix} * & * & * & \cdots & * \end{bmatrix}}_{n} \} 1$

An n x n matrix is a <u>square matrix</u>. $\left.\underbrace{\begin{bmatrix} * & * & \cdots \\ * & * & \\ \vdots & & \ddots & \\ & & & * \end{bmatrix}}_{n}\right\} n$

The entry in the i^{th} row and j^{th} column is called the <u>(i,j)-entry</u> of a matrix. If A is an $m \times n$ matrix, with (i,j)-entry a_{ij}, we write it as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$$

or, shorthand we simply write $A = [a_{ij}]$.

Addition and scalar multiplication

If two matrices have the same number of rows and columns we can add them by adding the corresponding entries.

So if $A = [a_{ij}]$, $B = [b_{ij}]$ are both $m \times n$ matrices, then

$$A + B = [a_{ij} + b_{ij}].$$

Ex: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 3 & 7 \end{bmatrix}$

If A is any matrix, and k is a real number, we define the <u>scalar multiple</u> kA to be the matrix obtained by multiplying each entry of A by k.

i.e. if $A = [a_{ij}]$, then $kA = [ka_{ij}]$.

Ex: $-2 \begin{bmatrix} 1 & 1 \\ 3 & -7 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -6 & 14 \end{bmatrix}.$

Transpose of a matrix

If A is an $m \times n$ matrix, the <u>transpose</u> of A, written A^T is the $n \times m$ matrix whose rows are the columns of A in the same order. So the (i,j)-entry of A is the (j,i)-entry of A^T.

Ex: If $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 4 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 3 & 4 \end{bmatrix}.$

A matrix is symmetric if it is equal to its transpose.

i.e. if $A = A^T$.

(Note that symmetric matrices must be square).

Ex: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 6 \end{bmatrix}$ is symmetric

← axis of symmetry

Practice problems: 2.1 : 1a, 2gh, 4, 14